

Critical Values of x (page 134) For a function, $f(x)$, a critical value is a value of x for which the function is defined and continuous and (a) $f'(x) = 0$ or (b) $f'(x)$ is undefined.

A point with a critical value for x may be a relative maximum, a relative minimum, or neither.

Examples:

$f(x) = 4 - x^2$	$f'(x) = -2x$	$f'(x) = 0$ when $x = 0$	relative max
$f(x) = x^3$	$f'(x) = 3x^2$	$f'(x) = 0$ when $x = 0$	nether min nor max
$f(x) = x^{2/3}$	$f'(x) = (2/3)x^{-1/3}$	$f'(x)$ is undefined when $x = 0$,	relative min

First Derivative Test (page 135). Given that c is a critical value for $y = f(x)$,

- (a) if $f'(x)$ changes from positive to negative at $x = c$, then $f(x)$ is changing from increasing to decreasing and the point $(c, f(c))$ is a **relative maximum**.
- (b) if $f'(x)$ changes from negative to positive at $x = c$, then $f(x)$ is changing from decreasing to increasing and the point $(c, f(c))$ is a **relative minimum**.
- (c) if $f'(x)$ does not change sign at $x = c$, then the point $(c, f(c))$ is a **neither** a relative maximum nor a relative minimum.

See Example 2 and Figure 4-36 on page 135.

Concavity (page 136): The curve $y = f(x)$ is concave upward when $f''(x) > 0$ and concave downward when $f''(x) < 0$.

Second Derivative Test (page 136)

- (a) If $f'(c) = 0$ and $f''(c) < 0$, then the curve is concave downward at $(c, f(c))$ and the point $(c, f(c))$ is a **relative maximum**.
- (b) If $f'(c) = 0$ and $f''(c) > 0$, then the curve is concave upward at $(c, f(c))$ and the point $(c, f(c))$ is a **relative minimum**.
- (c) If $f'(c) = 0$ and $f''(c) = 0$ then the Second Derivative Test fails. Use the first derivative test to determine whether you have a relative maximum, a relative minimum, or neither.

Point of inflection (page 136): Any point where the curve changes from concave upward to concave downward or from concave downward to concave upward is a point of inflection.

Examples 5 and 6 (page 138) show how to use the first and second derivatives of polynomials to find relative maxima and minima as well as points of inflection and to use these points to sketch the graph.

Examples 1-4 on pages 140-142 extend this analysis to rational functions. Here you will need to find the domain, vertical asymptotes and behavior as $x \rightarrow \infty$ and $x \rightarrow -\infty$.

Summary: What do values of a function and its first and second derivatives tell you about the graph?

When $f(x) = 0$, the graph crosses or touches the x -axis	When $f(x) > 0$, the curve lies above the x -axis	When $f(x) < 0$, the curve lies below the x -axis
When $f'(x) = 0$, we have a critical value. The point may be a relative maximum, a relative minimum, or neither	When $f'(x) > 0$, the value of the function is increasing as x increases.	When $f'(x) < 0$ the value of the function is decreasing as x increases.
When $f''(x) = 0$ we <u>may</u> have a point of inflection.(not necessarily)	When $f''(x) > 0$, the curve is concave upward.	When $f''(x) < 0$, the curve is concave downward.

Exercises: Use the above information to sketch the graphs of these functions. Identify all relative maxima and minima, points of inflection, and asymptotes. Then check your sketches using the calculator.

Page 139: 17, 19, 21, 23, 25, 27, 31, 35

Page 143: 3, 5, 9, 11, 15, 17